“…it's not the most important thing in your life right now. But what is important is gravity.”
Arnold Schwarzenegger as Colonel John Matrix, Commando (Check out this classic of American cinema!)

GEOS322 Lab 8: Gravity and Isostasy (35 points)

Group Exercises
1. Tidal and Drift Corrections for Gravity Measurements (4)
There are two fundamental corrections which must be done to all gravity measurements in order to arrive at a measured gravity value which can be interpreted in terms of subsurface density variations.

Tidal effects: Due to the gravitational attraction of the sun and the moon, the Earth experiences “tidal effects” in the gravity field at the Earth's surface. Besides the simple gravity field of the moon and sun affecting gravity at the surface, these gravity fields also produce "Earth tides" which are deformations of the solid Earth.

Earth tides have an amplitude of a little less than one meter. This means that during a 24 hour period, the surface of the Earth at any observing location will actually move a fraction of a meter closer to and then farther away from the center of mass of Earth, with resulting changes in gravity measured at the surface. These effects accumulate as a tidal influence on the gravity field. An example graph for a typical mid-latitude observing locality is shown in the figure below. Because we know the motions of the sun and moon with respect to the Earth with great precision, we can calculate the gravitational tidal effects to very high precision. These effects must be removed from measurements.

Instrument drift: The inner workings of a gravimeter are sufficiently complex that the instrument has an inherent amount of drift in measured values of gravity, even when measurements are taken at the same location and corrected for tidal effects. This drift is due to small changes in the spring "constant" of the gravimeter due to changes in temperature and movement of the gravimeter from one station to another during a
measuring session. The combined effects of tidal effects and instrument drift are illustrated in the schematic figure below.

![Schematic figure showing tidal variations and instrument drift](image)

The way we deal with this instrument drift is to measure gravity at an established "base station" at the start and end of any gravity measurement session. These base station readings are first corrected for tidal effects. Then the base station readings (corrected for tidal effects) are used to determine the drift rate which occurred during the measurement session. To correct a gravity reading for both tidal effects and instrument drift, the procedure is to: (1) subtract out the tidal effect (usually determined by a computer program which contains all the orbital parameters of sun and moon motion with respect to the Earth); then (2) correct for instrument drift by interpolating the drift correction at the time of the gravity measurement between the two base station measurements.

The table below displays gravity measurements from a small survey. The two base readings are in bold and measurements elsewhere are in between; use it to work through the tidal and drift corrections which must be made to the measured values of gravity. First determine the gravity measurements corrected for tidal effects and then make the correction for instrument drift. Fill this information in to the shaded columns in the table.
<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Tidal Effect (mGal)</th>
<th>Base Reading (mGal)</th>
<th>Reading (mGal)</th>
<th>Tidal Corrected (mGal)</th>
<th>Drift Corrected (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>-0.030</td>
<td>5141.620</td>
<td>5141.620</td>
<td>5141.650</td>
<td>5141.650</td>
</tr>
<tr>
<td>15</td>
<td>-0.050</td>
<td></td>
<td>5141.210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>-0.055</td>
<td></td>
<td>5141.715</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>-0.070</td>
<td></td>
<td>5141.760</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>-0.075</td>
<td></td>
<td>5141.785</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>-0.065</td>
<td></td>
<td>5141.815</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-0.050</td>
<td></td>
<td>5141.820</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
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<td></td>
<td>5141.825</td>
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<tr>
<td>22</td>
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<td></td>
<td>5141.795</td>
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<td></td>
</tr>
<tr>
<td>23</td>
<td>0.065</td>
<td>5141.805</td>
<td>5141.805</td>
<td>5141.740</td>
<td>5141.650</td>
</tr>
</tbody>
</table>

Compared to the subsequent free-air correction (~0.3 mGal/m), the tide and drift corrections are small and might seem insignificant.

**Q:** Depending on the scale and sampling density of a gravity survey, the resolution of gravity measurements can vary considerably. An example resolution from a study by Blakely et al. 1996 is 0.05 mGal. Explain why proper tide and drift corrections are important for their study.

2. The Free-Air Correction (4)
As discussed in lecture, we must account for variations in the observed gravitational acceleration due to elevation of the gravity station. We do this by using a *free air correction*. To a first order approximation, the free-air correction is -0.3086 mGals per meter of station elevation above sea level. The minus sign indicates that as the elevation increases, the observed acceleration decreases. The magnitude of the number says that if two gravity readings are made at the same location, but one is done a meter above the other, the reading taken at the higher elevation will be 0.3086 mGal less than the reading taken at the lower elevation.

An ideal way to measure the free-air effect would be to measure gravity at successively higher locations in a tower. We can do the next best thing by measuring gravity on
successively higher floors of a tall building like the Gould/Simpson Building. We have done this and the results are shown on the graph below.

Note that the ground floor point should be considered as an outlier. Using the plot of gravity measurements in the Gould-Simpson Building, determine the vertical gradient of measured gravity \((dg/dz)\) by fitting a line through the measured gravity values and determining the slope of that line.

**Q:** What value do you derive for the vertical gradient of measured gravity? List your result in mGals/m.
Individual Exercises

3. Isostasy (20 pts)
In this part of the lab you will practice working with equations for Airy and Pratt-style isostasy in order to understand how mountain ranges are gravitationally compensated. The following gives a review of the equations that you will need to understand and use.

Both Pratt and Airy isostasy use a compensation depth, which is the depth in the mantle at which pressure does not change in the horizontal direction. The lithostatic pressure exerted by a uniform column of crust and/or mantle:

\[ P = \rho gh \]  

where \( P \) is lithostatic pressure, \( \rho \) is rock density, \( g \) is gravity, and \( h \) is column’s height.

**Pratt isostasy** acts on the principle that if elevation changes between crustal columns, then one can isostatically balance (or "compensate") the columns by changing \( \rho \) for each column of crust by an appropriate amount. The following equation describes this:

\[ \frac{P}{g} = \rho_1 h_1 = \rho_2 h_2 = \rho_3 h_3 = \ldots etc… \]  

where \( \rho_1, \rho_2, \ldots \) are different crustal densities. Pratt isostasy assumes that the depth each column extends to (the depth of compensation) remains constant (i.e. a flat Moho).

**Airy isostasy** compensates for differing elevations between crustal columns by varying only \( h \) while \( \rho \) stays the same for the crust for all columns. The following equation shows this:

\[ \frac{P}{g} = \rho_c h_1 + \rho_m h_1' = \rho_c h_2 + \rho_m h_2' = \rho_c h_3 + \rho_m h_3' = \ldots etc… \]  

where \( \rho_c \) is the density of crust, \( \rho_m \) is the density of mantle, and \( h_x' \) is the height of mantle under a column of crust of thickness \( h_x \). In this case, the depth each column extends to changes (i.e. a changing Moho or "crustal root"). Because of this, mantle and mantle densities are also involved in the calculation. The depth of compensation used in balancing columns can change depending on which columns you are using.

Now, imagine a hypothetical mountain range made of 3 columns of rock, each isostatically isolated from each other. A topographic survey along a N-S transect shows the average elevation of the highest crustal block is 5 km. The crustal block to the north has an average elevation of 3 km, and the block to the south has an average elevation of 1 km. You also know that this mountain range is in isostatic equilibrium with a column of crust on the adjoining plains. From geophysical surveys on the plains, you know the crust is 35 km thick, with a density of 2800 kg/m\(^3\). You also know that the mantle under the plains has a density of approximately 3300 kg/m\(^3\). The elevation of the plain is exactly at sea level (but it isn't flooded!).
3.1 Pratt-style Isostasy

![Diagram of Pratt-style Isostasy]

**a.** On the diagram above, draw in the plains crustal column under the arrow, labeling the crust and the mantle with the appropriate densities.

**b.** For a **Pratt** model of isostatic compensation, what is the depth of compensation? Mark and label this on the diagram above and draw in the columns under the mountains.

**c.** Using the depth of compensation from part **b** and equation (2), calculate the densities of each crustal column needed in order to isostatically compensate the mountains according to the **Pratt**-style of isostasy. Label each mountain column with the density you calculated and show your work!

**d. BONUS (3 pts.)** Draw the approximate bouguer anomaly for measurements taken across the survey area.
3.2 Airy-style Isostasy

a. Above is the same diagram you started with in 4.1.a above. If you tried to compensate the mountains with Airy-style isostasy, which column would have the deepest corresponding Moho?

b. You will use the column identified in a to make your first Airy isostatic calculation assuming the depth of compensation is at the base of that column. If it is balanced with the plains column, using equation (3) above, we can set up an equation describing the equality of $P/g$ for each column:

$$\rho_c h_p + \rho_m h'_p = \rho_c h_m$$ (4)

For each quantity in this equation, give a description (on the next page) of the variable and the corresponding quantity given in the introduction to this problem. There should be two variables of unknown quantity—label these with a "?".

$$\rho_c =$$
$$h_p =$$
$$\rho_m =$$
$$h'_p =$$
$$h_m =$$
One of the unknown quantities, $h_m$, can be further broken down into several quantities that we do know. First, we know the height of the part of the column of crust above sea level, which we will call $h_e$. The remaining part of the column is just the part between sea level and the compensation depth. Since the compensation depth for the mountain column and the plains column is the same, we can rewrite this part in terms of the variables we have defined for the plains column. Therefore, we can use the following expression to redefine $h_m$:

$$h_m = h_e + h_p + h'_p \quad (5)$$

The schematic diagram below illustrates this concept.

\[\text{c. Replace the term } h_m \text{ in equation (4) with the quantity described by equation (5) and write the result below.}\]

\[\text{d. Now we only have one unknown variable: } h'_p. \text{ Solve the equation you wrote in c for this variable and write it in the space below.}\]
e. Use the quantities given to you in this problem to calculate a value for \( h_p' \). Then use equation (5) to calculate the height of the crustal column (\( h_m \)) necessary to support the highest part of the mountains with Airy-style isostasy. Sketch the resulting crustal column on the diagram in part a above.

f. Now use the same method developed in a-e above to calculate the height of the crustal columns necessary to support the other parts of the mountain range. Write all equations used and intermediate quantities calculated (attach a separate sheet of needed. Sketch the resulting crustal columns on the diagram in part a above.

g. BONUS (3 pts.) Draw the approximate bouguer anomaly for measurements taken across the survey area.

3.3 Airy Isostasy and Erosion
Imagine a mountainous column of crust with average elevation of 5 km in Airy Isostatic balance with a plains crustal column (the situation in 4.2.a-e). We all know that high mountains erode like crazy! Our hypothetical mountain range develops a river system which erodes away 5 km off the top of the mountain range. The crustal column will compensate by "popping up" similar to an ice cube in your soda glass, which pops up when you stop holding it down with your finger. **Calculate the new elevation of the highest part of the mountain range, to the meter (or thousandths of km's) after it has isostatically re-compensated itself after the erosion.** Hint: start by figuring out the new thickness of the crust under the mountain range (\( h_m \)) and then refer back to equation (4). Remember that your compensation depth will change as the crust pops up!
4. Gravity Anomalies (7)
A vast blanket of ice covers Antarctica (at least for a little while longer). The ice has a density of 0.92 g/cm$^3$. At a location of 80° S, 120° W, the ice surface is 1525 m above sea level, and the ice layer is 2470 m thick. Here the value of observed gravity is 983061 mGals. Density of rock beneath ice is assumed to be 2.67 g/cm$^3$. Sketch this situation in cross-section. Calculate the free-air and Bouguer gravity anomalies. Explain why Bouguer anomalies are usually negative over mountains and positive over oceans.